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Effect of collisions on wave motions in a plasma with anisotropic pressure

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Abstract. The development with time of small-amplitude oscillations, having harmonic spatial dependence, in a magnetoactive plasma is examined with the aid of moment equations. The plasma, which is assumed to be homogeneous and unbounded, has an initial anisotropic distribution of pressure. Neglecting the motion of the ions, we consider in detail the effect of collisions on the further development of the system for the propagation vector along or perpendicular to the magnetic field. For the collisionless case we obtain the dispersion relations derived by Jaggi. For the case of weak collisions explicit expressions for the damping and phase shift coefficients are evaluated. It is found that, for longitudinal propagation along the magnetic field, the collisional damping decreases or increases according to whether the perpendicular pressure is greater or less than the longitudinal pressure. High-frequency transverse waves may exhibit a collision-induced instability if the pressure anisotropy is large and of the correct sign. For extremely-high-pressure relaxation frequencies we recover the ordinary dispersion relations, in which only the momentum relaxation frequency contributes to the damping of the wave.

1. Introduction

The subject of waves and instabilities in a plasma is of extreme interest because of its extensive applications to laboratory devices, astrophysical systems and thermonuclear fusion. The literature on the propagation of waves in a plasma is very extensive. However, sufficient attention has not been paid to the problem of wave motions in a plasma with anisotropic pressure. A magnetoactive plasma can support anisotropic pressure when the collision or collision-like relaxation frequencies are sufficiently small. The collisions, besides restoring the isotropy of pressure, may lead to new instabilities which may be of great significance for thermonuclear devices where a high-density hot plasma is confined by the mechanism of adiabatic magnetic compression. These instabilities may also be of interest in the study of the magnetoactive regions of the astrophysical systems.

Jaggi (1962) has considered wave motions in a plasma with anisotropic pressure, in which the effect of pressure anisotropy on the propagation characteristics of the waves was discussed. Collisions were completely neglected in this investigation. Experimental investigations of the effect of pressure anisotropy on instabilities have been made by Post and Perkins (1961). The author (Sharma 1967) has examined the effect of collisions on the propagation of waves in a plasma with a fixed anisotropic pressure. It was assumed here that the collisions forced the perturbed pressure tensor to relax to the equilibrium value.

In the present investigation we consider an electron plasma system in which the collisions are initiated after it has developed a certain pressure anisotropy. The subsequent development of the system is examined with the aid of moment equations which include the momentum and pressure relaxation frequencies. For the collisionless case we obtain Jaggi's dispersion relations. If the relaxation frequencies are much smaller than the propagation frequency, the collisions slowly restore the pressure isotropy and may lead to new instabilities in the system. The conditions for these instabilities are deduced. Explicit expressions are derived for the phase shift and damping coefficients. If the pressure relaxation frequency is extremely high, the pressure anisotropy is promptly destroyed by the collisions. In this situation we readily recover the usual dispersion relations; here damping results from the momentum relaxation collisions only. It is also found in general that the momentum relaxation mechanism always produces damping; the pressure relaxation mechanism, on the other hand, contributes to damping or instability, depending upon the situation and the value of the anisotropy parameter α.

Two cases have been considered in detail: that where the propagation vector is directed along the magnetic field, and that where the propagation vector is directed perpendicular to the magnetic field.

2. Basic equations

Starting with the Boltzmann transport equation with a simple collision term in which the collisions force the distribution function to relax to the equilibrium distribution function, we obtain the following moment equations for the electron fluid:

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x_j}(nu_j) = 0 \tag{2.1}$$

$$\frac{du_i}{dt}+\frac{\partial}{\partial x_j}p_{ij}+\frac{e}{m}\left(E_i+\frac{1}{c}\epsilon_{ijk}u_jB_k\right)+\nu u_i = 0 \tag{2.2}$$

$$\frac{dp_{jk}}{dt}+\frac{\partial q_{ijk}}{\partial x_i}+\frac{\partial u_i}{\partial x_i}p_{jk}+\frac{\partial u_k}{\partial x_i}p_{ji}+\frac{\partial u_j}{\partial x_i}p_{ik} +\frac{eB_l}{mc}(\epsilon_{jnl}p_{kn}+\epsilon_{knl}p_{jn}) = -\nu'(p_{jk}-\delta_{jk}p). \tag{2.3}$$

In the above equations n, m and e are the number density, mass and magnitude of charge of the electron, respectively; u_i, p_{ij} and q_{ijk} are the components of the fluid velocity, pressure tensor and the heat flow tensor, respectively. p is $\frac{1}{3}$(trace of the pressure tensor). E_i and B_i are the electric and magnetic fields, respectively. ϵ_{ijk} is the Levi–Civita tensor density and δ_{jk} is the Kronecker delta. ν is the momentum relaxation frequency, and is roughly equal to the number of collisions per unit time which an electron makes with ions or neutrals; ν' may be called the pressure relaxation frequency, and is equal to the number of collisions per unit time which an electron makes with other electrons, ions and neutrals. Thus ν' includes the electron–electron collisions, which are effective in restoring the isotropy of pressure. The electromagnetic fields are governed by Maxwell's equations:

$$\epsilon_{ijk}\frac{\partial E_k}{\partial x_j}+\frac{1}{c}\frac{\partial B_i}{\partial t} = 0 \tag{2.4}$$

$$\epsilon_{ijk}\frac{\partial B_k}{\partial x_j} = \frac{1}{c}\frac{\partial E_i}{\partial t}-\frac{4\pi enu_i}{c} \tag{2.5}$$

$$\frac{\partial E_j}{\partial x_j} = 4\pi e(n_0-n) \tag{2.6}$$

$$\frac{\partial B_j}{\partial x_j} = 0 \tag{2.7}$$

where n_0 is the equilibrium number density of ions or electrons.

In order to obtain a closed set of equations, we neglect the higher-order moments in equation (2.3), viz. the divergence of the heat flow tensor. Equations (2.1)–(2.7) then form a closed set suitable for the investigation of the problem.

3. Linearization and treatment of pressure tensor

We consider an unbounded, homogeneous, quasi-neutral, stationary plasma embedded in a uniform magnetostatic field. It is assumed that the plasma has initially a certain pressure anisotropy and the collisions are started at zero time. The subsequent behaviour of the system is considered in the present analysis. We neglect ion motion. All the equations (2.1)–(2.7) are linearized about the initial static state in the usual manner. There is no unperturbed electric field in the initial state.

It is convenient to separate the components of the pressure tensor into two parts: a slowly varying and spatially independent part P_{jk}, and rapidly oscillating part $\hat{p}_{jk}$. Hence we write

$$p_{jk} = \hat{p}_{jk} + P_{jk}. \tag{3.1}$$

Using (3.1), we obtain from equation (2.3)

$$P_{11} = P_{22} = p_0 + p^* \exp(-\nu' t) \tag{3.2}$$

$$P_{33} = p_0 - 2p^* \exp(-\nu' t) \tag{3.3}$$

$$P_{12} = P_{13} = P_{23} = 0 \tag{3.4}$$

where p_0 is the isotropic pressure which will be restored by the collisions after a very long time, and p^* determines the initial pressure anisotropy of the system. In deriving equations (3.2) and (3.3) we have used the fact that the scalar pressure is not affected by the collisions. The linearized pressure tensor equation can be written as

$$\frac{\partial \hat{p}_{jk}}{\partial t} + P_{jk}\frac{\partial u_i}{\partial x_i} + \frac{\partial u_j}{\partial x_i}P_{ik} + \frac{\partial u_k}{\partial x_i}P_{ji} + \frac{eb_l}{mc}(\epsilon_{jnl}P_{kn} + \epsilon_{knl}P_{jn})$$
$$+ \frac{eB_l}{mc}(\epsilon_{jnl}\hat{p}_{kn} + \epsilon_{knl}\hat{p}_{jn}) + \nu'(\hat{p}_{jk} - \delta_{jk}\hat{p}) = 0 \tag{3.5}$$

where b_l is the perturbed magnetic field and u_i is the perturbed fluid velocity; B_l is the static uniform magnetic field B_0 which we take along the z axis. It will be noted that the separation of the pressure tensor into two components, as given by equation (3.1), is possible since the time scales of variation of P_{jk} are quite different from those of the terms of equation (3.5). Taking the propagation vector in the xz plane, we assume that the variables have harmonic spatial dependence of the form $\exp(ik_1x + ik_3z)$, where k_1 and k_3 are the components of the propagation vector along the x and z axes respectively.

The perturbed magnetic and electric fields are governed by the following relations:

$$\mathscr{D}_{\mathrm{b}}\mathbf{b} = -4i\pi e n_0 c\mathbf{k} \times \mathbf{u} \tag{3.6}$$

and

$$\mathscr{D}_{\mathrm{b}}\frac{\partial}{\partial t}\mathbf{E} = 4\pi e n_0\left\{\frac{\partial^2}{\partial t^2}\mathbf{u} + c^2\mathbf{k}(\mathbf{k}\,.\,\mathbf{u})\right\} \tag{3.7}$$

where the operator

$$\mathscr{D}_{\mathrm{b}} = \frac{\partial^2}{\partial t^2} + k^2c^2. \tag{3.8}$$

Using the above equations, we obtain the following relations for the various components of the pressure tensor:

$$\mathscr{D}_{12}\mathscr{D}'\hat{p}_{11} + i\mathscr{D}_{12}\{(3k_1\dot{u}_1 + k_3\dot{u}_3)(p_0 + p')\} + i\nu'\mathscr{D}_{12}\{\tfrac{5}{3}p_0(k_1u_1 + k_3u_3) - \tfrac{7}{3}(k_1u_1 + k_3u_3)p'\}$$
$$+ 4i\Omega^2\{-k_1\dot{u}_1(p_0 + p') + \nu' k_1u_1p'\} + 2i\Omega\{-k_1\ddot{u}_2(p_0 + p') + \nu' k_1\dot{u}_2(-p_0 + p')\} = 0 \tag{3.9}$$

$$\mathscr{D}_{12}\hat{p}_{12} + 2i\Omega k_1u_1(p_0 + p') + ik_1\dot{u}_2(p_0 + p') + ik_1u_2\nu' p_0 = 0 \tag{3.10}$$

$$\mathscr{D}_{\mathrm{b}}\mathscr{D}_{13}\hat{p}_{13} + i\mathscr{D}_{\mathrm{b}}\{(k_1\dot{u}_3 + k_3\dot{u}_1)p_0 + \nu'(k_1u_3 + k_3u_1)p_0 + (k_1\dot{u}_3 - 2k_3\dot{u}_1)p'\}$$
$$- i\Omega\mathscr{D}_{\mathrm{b}}(k_3u_2p_0 - 2k_3u_2p') - 3i\omega_{\mathrm{p}}{}^2p'(k_3\dot{u}_1 - k_1\dot{u}_3 - \Omega k_3u_2) = 0 \tag{3.11}$$

$$\mathscr{D}_{12}\mathscr{D}'\hat{p}_{22} + i\mathscr{D}_{12}\{(k_1\dot{u}_1 + k_3\dot{u}_3)(p_0 + p')\} + i\nu'\mathscr{D}_{12}\{\tfrac{5}{3}p_0(k_1u_1 + k_3u_3)$$
$$- \tfrac{1}{3}(k_1u_1 + 7k_3u_3)p'\} - 4i\Omega^2\{-k_1\dot{u}_1(p_0 + p') + \nu' k_1u_1p'\}$$
$$+ 2i\Omega\{k_1\ddot{u}_2(p_0 + p') + \nu' p_0k_1\dot{u}_2 - \nu' p' k_1\dot{u}_2\} = 0 \tag{3.12}$$

$$\mathscr{D}_b \mathscr{D}_{13} \hat{p}_{23} + i\mathscr{D}_b(k_3 \dot{u}_2 p_0 + \nu' k_3 u_2 p_0 - 2k_3 \dot{u}_2 p') + i\Omega \mathscr{D}_b\{(k_1 u_3 + k_3 u_1)p_0 + (k_1 u_3 - 2k_3 u_1)p'\} - 3i\omega_p^2 p'\{\Omega(k_3 u_1 - k_1 u_3) + k_3 \dot{u}_2\} = 0 \tag{3.13}$$

$$\mathscr{D}' \hat{p}_{33} + i(k_1 \dot{u}_1 + 3k_3 \dot{u}_3)(p_0 - 2p') + i\nu'\{\tfrac{5}{3}(k_1 u_1 + k_3 u_3)p_0 + \tfrac{1}{3}(8k_1 u_1 + 14k_3 u_3)p'\} = 0. \tag{3.14}$$

Taking the trace of the pressure tensor, we obtain

$$\frac{\partial \hat{p}}{\partial t} + i\{\tfrac{5}{3}(k_1 u_1 + k_3 u_3)p_0 + \tfrac{2}{3}(k_1 u_1 - 2k_3 u_3)p'\} = 0 \tag{3.15}$$

where

$$\omega_p^2 = 4\pi n_0 \frac{e^2}{m}, \qquad \Omega = \frac{eB_0}{mc}, \qquad p' = p^* \exp(-\nu' t)$$

and the operators are defined as

$$\mathscr{D}_{12} = \frac{\partial^2}{\partial t^2} + 2\nu' \frac{\partial}{\partial t} + \nu'^2 + 4\Omega^2; \qquad \mathscr{D}_{13} = \frac{\partial^2}{\partial t^2} + 2\nu' \frac{\partial}{\partial t} + \nu'^2 + \Omega^2;$$

$$\mathscr{D}' = \frac{\partial^2}{\partial t^2} + \nu' \frac{\partial}{\partial t}. \tag{3.16}$$

$\dot{u}$ denotes the time derivative. Using the above relations in the linearized equation of motion, we obtain a set of three linear coupled differential equations, which in principle can be solved to obtain the three components of the perturbed velocity as functions of time. For a general direction of the wave vector the expressions become extremely complicated. The situation is very much simplified when the propagation vector is in the direction of the magnetic field or perpendicular to it.

4. Propagation vector along the magnetic field

When the propagation vector is directed along the magnetic field ($k_1 = 0$, $k_3 = k$) the components of the perturbed velocity along and perpendicular to the magnetic field become decoupled. The longitudinal component of the perturbed velocity is governed by the equation

$$\dddot{u}_3 + (\nu + \nu')\ddot{u}_3 + \dot{u}_3[\omega_p^2 + \nu\nu' + 3k^2 s^2\{1 - 2\alpha \exp(-\nu' t)\}] + u_3 \nu'[\omega_p^2 + k^2 s^2\{\tfrac{5}{3} + \tfrac{14}{3}\alpha \exp(-\nu' t)\}] = 0 \tag{4.1}$$

where $\alpha = p^*/p_0$ is the anisotropy parameter; $s = (p_0/mn_0)^{1/2}$ is the isothermal velocity of sound. Equation (4.1) has been derived previously (Sharma, to be published). We shall briefly reproduce the important results. In the absence of collisions, we obtain the dispersion relation for electron plasma waves with the effective value of γ equal to 3:

$$\omega^2 = \omega_p^2 + 3k^2 s^2(1 - 2\alpha). \tag{4.2}$$

In the presence of weak collisions ($\nu' \ll \omega$) equation (4.1) can be solved to give

$$u_3 = u_{30} \exp\left\{i\omega\left(1 + \frac{3\alpha\nu' k^2 s^2 t}{2\omega^2}\right)t - \left(\frac{\nu}{2} + \frac{4 - 5\alpha}{6}\frac{k^2 s^2 \nu'}{\omega^2}\right)t\right\} \tag{4.3}$$

where ω is given by equation (4.2). Equation (4.3) shows that in the absence of anisotropic pressure both ν and ν' contribute to the damping of the wave. However, the wave suffers an instability only if

$$\alpha > \frac{4}{5} + \frac{3\nu\omega^2}{5\nu' k^2 s^2}. \tag{4.4}$$

However, this is not possible since α must lie between -1 and $\frac{1}{2}$. The damping decreases if $\alpha > 0$, i.e. when the perpendicular pressure is larger than the longitudinal pressure. In the opposite extreme case of very-high-pressure relaxation frequency, equation (4.1) reduces to an ordinary differential equation with constant coefficients, and we obtain a dispersion relation

$$\omega^2 = \omega_p{}^2 + \tfrac{5}{3}k^2s^2 + i\omega\nu \tag{4.5}$$

where u_3 varies as $\exp(i\omega t)$. Equation (4.5) indicates that only ν contributes to the damping of the wave, and the effective value of γ now becomes equal to $\frac{5}{3}$.

The components of the perturbed velocity perpendicular to the magnetic field are governed by the equation

$$\begin{aligned}\mathscr{D}_b\mathscr{D}_{13}(\dot{u}+\nu u) + k^2s^2\mathscr{D}_b(\dot{u}+\nu' u - 2\dot{u}\alpha') + \omega_p{}^2\mathscr{D}_{13}\dot{u} - 3k^2s^2\omega_p{}^2\alpha'\dot{u}\\ \mp i\Omega[\mathscr{D}_{13}\mathscr{D}_b u + 3\alpha' k^2s^2\omega_p{}^2 u - k^2s^2\mathscr{D}_b\{u(1-2\alpha')\}] = 0\end{aligned} \tag{4.6}$$

where $u = u_1 \pm iu_2$ and $\alpha' = \alpha\exp(-\nu' t)$. The upper and lower signs refer to the clockwise and anticlockwise modes of circular rotation. In the absence of collisions equation (4.6) yields the dispersion relation

$$\begin{aligned}1 - \frac{k^2s^2(1-2\alpha)}{\omega^2-\Omega^2} - \frac{\omega_p{}^2}{\omega^2-k^2c^2} - \frac{3\alpha k^2s^2\omega_p{}^2}{(\omega^2-\Omega^2)(\omega^2-k^2c^2)}\\ \mp\frac{\Omega}{\omega}\left\{1+\frac{k^2s^2(1-2\alpha)}{\omega^2-\Omega^2}+\frac{3\alpha\omega_p{}^2k^2s^2}{(\omega^2-\Omega^2)(\omega^2-k^2c^2)}\right\} = 0.\end{aligned} \tag{4.7}$$

Equation (4.7) is the same as the one obtained by Jaggi (1962). In the case of a cold plasma equation (4.7) becomes

$$\omega^2 - k^2c^2 = \frac{\omega\omega_p{}^2}{\omega\mp\Omega}. \tag{4.8}$$

In the presence of weak collisions ($\nu' \ll \omega$) the pressure anisotropy is gradually reduced. In order to investigate the effect of these collisions on the propagation of waves we consider the system for a time t, such that

$$\nu' t \ll 1 \quad \text{and} \quad \omega t \gg 1. \tag{4.9}$$

The first condition ensures that the effect of collisions will remain small, and the second condition makes it possible to consider several cycles of wave propagation. Under these conditions u has a solution of the type

$$u = u_0\exp\{i(\omega+a)t + bt^2\} \tag{4.10}$$

where a and b are small quantities and ω is the characteristic frequency of propagation in the absence of collisions. It is seen that the imaginary part of a determines the coefficient of damping or instability, depending upon its sign. $-ibt$, if real, gives the resulting phase shift caused by the collisions and pressure anisotropy. a and b can be determined by substituting equation (4.10) in equation (4.6). The expressions so obtained are still very complicated. Hence we shall restrict ourselves to the case when the phase velocity of the wave is much larger than the velocity of sound of the electron fluid, i.e. $\omega \gg ks$. Considering the case when $\Omega \to 0$, we obtain

$$ia = \frac{\nu' s^2(\omega^2-\omega_p{}^2)}{4\omega^4c^2}\{\alpha(3\omega_p{}^2+8\omega^2) - 2\omega_p{}^2\} - \frac{\nu\omega_p{}^2}{2\omega^2} \tag{4.11}$$

and

$$b = -\frac{i\alpha\nu' s^2(\omega^2-\omega_p{}^2)\omega_p{}^2}{4\omega^3c^2}. \tag{4.12}$$

It is seen from equation (4.11) that ν produces damping in the wave. The pressure relaxation mechanism, for $\omega > \omega_p$, reduces or increases damping depending upon whether

α is positive or negative. The system can undergo an instability if (for $\omega \gg \omega_p$)

$$\alpha > \frac{\omega_p^2}{4\omega^2}\left(\frac{\nu c^2}{\nu' s^2} + 1\right). \tag{4.13}$$

This shows that a large pressure anisotropy is necessary for the instability to occur. The phase shift term vanishes in the absence of ν' or α.

In the other limit when the cyclotron frequency Ω is much larger than the propagation frequency ω, we obtain

$$b = \mp \frac{3i\alpha\nu'\omega_p^2 k^2 s^2}{2\Omega(\omega_p^2 \pm 2\omega\Omega)} \tag{4.14}$$

and

$$ia = \frac{\alpha\nu'\Omega^4 k^2 s^2(8\omega^2 + 3\omega_p^2)}{D^2} \mp \frac{\nu\omega\Omega\omega_p^2}{D} \mp \frac{\nu'\omega k^2 s^2 \omega_p^2}{\Omega D} \tag{4.15}$$

where

$$D = \Omega^2(\omega_p^2 - 2\omega^2) \pm \omega\Omega(2\Omega^2 + \omega_p^2).$$

If $\omega \gg \omega_p$, the above equations are simplified to yield

$$b = -\frac{3i\alpha\nu'\omega\omega_p^2 s^2}{4\Omega^2 c^2} \tag{4.16}$$

and

$$ia = \frac{2\alpha\omega^2 s^2}{\Omega^2 c^2}\nu' - \left(\frac{\nu}{2} + \frac{s^2\nu'\omega^2}{2c^2\Omega^2}\right)\frac{\omega_p^2}{\Omega^2}. \tag{4.17}$$

From equation (4.17) it is obvious that an instability is possible for a large positive value of α which satisfies the inequality

$$\alpha > \frac{\omega_p^2}{4\omega^2}\left(\frac{\nu c^2}{\nu' s^2} + \frac{\omega^2}{\Omega^2}\right). \tag{4.18}$$

A comparison of equations (4.13) and (4.18) shows that for the instability to occur α must be positive, i.e. the perpendicular thermal energy must be larger than the longitudinal thermal energy.

If the pressure relaxation frequency ν' is much higher than all the frequencies of interest, equation (4.6) can be very much simplified. As $\nu' \to \infty$, equation (4.6) gives the dispersion relation

$$\omega^2 = k^2c^2 + \frac{\omega\omega_p^2}{\omega - i\nu \mp \Omega}. \tag{4.19}$$

Equation (4.19) is the usual expression for transverse waves propagating along the magnetic field. This expression is independent of the thermal effects. It is also seen that only the momentum relaxation frequency ν contributes to the damping of the wave.

5. Propagation vector perpendicular to the magnetic field

If the propagation vector is taken along the x axis while the magnetic field is pointing in the z direction, the equations of motion break into two independent sets: one of these is a differential equation which describes the behaviour of u_3, the component of the perturbed velocity along the magnetic field; the other set is composed of two simultaneous differential equations which govern the other two components, u_1 and u_2, of the perturbed velocity. The first of these can be expressed as

$$\mathscr{D}_b\mathscr{D}_{13}\left(\frac{\partial}{\partial t} + \nu\right)u_3 + k^2s^2[\mathscr{D}_b\{\dot{u}_3 + \nu' u_3 + \alpha\dot{u}_3\exp(-\nu' t)\}$$
$$+ 3\alpha\omega_p^2\dot{u}_3\exp(-\nu' t)] + \omega_p^2\mathscr{D}_{13}\dot{u}_3 = 0. \tag{5.1}$$

In the absence of collisions equation (5.1) gives the dispersion relation

$$1+\frac{k^2s^2(1+\alpha)}{\Omega^2-\omega^2}+\frac{3\alpha\omega_p{}^2k^2s^2}{(k^2c^2-\omega^2)(\Omega^2-\omega^2)}+\frac{\omega_p{}^2}{k^2c^2-\omega^2}=0 \tag{5.2}$$

which in the zero-temperature approximation reduces to

$$\omega^2=\omega_p{}^2+k^2c^2. \tag{5.3}$$

In the presence of weak collisions we consider the system for a time t, such that the inequality (4.9) is satisfied. We again assume that the time dependence of u_3 is of the form

$$u_3=u_{30}\exp\{i(\omega+a)t+bt^2\} \tag{5.4}$$

where $a \ll \omega$ and $b \ll 1$. Substituting (5.4) in equation (5.1) and retaining only the first-order small quantities, we see that the characteristic frequency of propagation is given by equation (5.2), with

$$b=\frac{i\nu'\alpha k^2s^2(k^2c^2+3\omega_p{}^2-\omega^2)}{4\omega A} \tag{5.5}$$

where

$$A=2\omega^2-\{k^2c^2+\Omega^2+k^2s^2(1+\alpha)+\omega_p{}^2\}$$

and

$$-ia=\frac{\nu(\omega^2-k^2c^2)(\omega^2-\Omega^2)}{2A\omega^2}+\frac{\nu'\alpha k^2s^2(k^2c^2+3\omega_p{}^2-\omega^2)}{A^2}+\frac{\nu'\alpha k^2s^2(3k^2c^2+9\omega_p{}^2+\omega^2)}{4A\omega^2}$$
$$+\frac{\nu'}{2A\omega^2}\{(2\omega^2-k^2s^2)(\omega^2-k^2c^2)-2\omega^2\omega_p{}^2\}. \tag{5.6}$$

As the phase velocity of the wave is much higher than the electron thermal velocity, $\omega \gg ks$, the above equations may be simplified to yield

$$b=\frac{i\alpha\nu'\omega_p{}^2(\omega^2-\omega_p{}^2)s^2}{2\omega(\omega^2-\Omega^2)c^2} \tag{5.7}$$

and

$$ia=-\frac{\nu\omega_p{}^2}{2\omega^2}-\frac{2\nu'\alpha s^2(\omega^2-\omega_p{}^2)}{c^2(\omega^2-\Omega^2)}\frac{2\omega^2+3\omega_p{}^2}{4\omega^2}+\frac{\nu'\omega_p{}^2(\omega^2-\omega_p{}^2)s^2}{2\omega^2(\omega^2-\Omega^2)c^2}\frac{\omega^2+\Omega^2}{\omega^2-\Omega^2}. \tag{5.8}$$

If $\omega \gg \omega_p$, we obtain, as $\Omega \to 0$,

$$b=\frac{i\alpha\nu'\omega_p{}^2s^2}{2\omega c^2} \tag{5.9}$$

and

$$ia=-\frac{\nu\omega_p{}^2}{2\omega^2}-\frac{\nu'\omega_p{}^2s^2}{2\omega^2c^2}-\frac{\nu'\alpha s^2}{c^2}. \tag{5.10}$$

For the instability to occur α must be large and negative, and must satisfy the inequality

$$-\alpha>\frac{\omega_p{}^2}{2\omega^2}\left(\frac{\nu c^2}{\nu's^2}+1\right). \tag{5.11}$$

If, on the other hand, $\Omega \gg \omega \gg \omega_p$, from equations (5.7) and (5.8) we have

$$b=-\frac{i\alpha\nu'\omega_p{}^2s^2\omega}{2\Omega^2c^2} \tag{5.12}$$

and

$$ia=-\frac{\nu\omega_p{}^2}{2\omega^2}-\frac{\nu'\omega_p{}^2s^2}{2\Omega^2c^2}+\frac{\nu'\alpha s^2\omega^2}{c^2\Omega^2}. \tag{5.13}$$

The condition for the instability now becomes

$$\alpha > \frac{\omega_p^2}{2\omega^2}\left(\frac{\nu c^2\Omega^2}{\nu' s^2\omega^2}+1\right). \tag{5.14}$$

If the pressure relaxation frequency is very high, we obtain from equation (5.1), as $\nu' \to \infty$, the dispersion relation

$$\omega^2 = k^2c^2+\frac{\omega\omega_p^2}{\omega - i\nu}. \tag{5.15}$$

Equation (5.15) represents the usual expression for the transverse waves in a plasma (Spitzer 1956).

Finally, we consider the case of coupled longitudinal–transverse waves which are described by a pair of coupled differential equations governing the behaviour of u_1 and u_2. These equations are

$$\begin{aligned}\mathscr{D}_{12}\left(\frac{\partial}{\partial t}+\nu'\right)\left(\frac{\partial}{\partial t}+\nu\right)\dot{u}_1 &+ k^2s^2[\mathscr{D}_{12}\{3\dot{u}_1(1+\alpha')+\tfrac{5}{3}\nu' u_1(1-\tfrac{4}{5}\alpha')\}]\\ &+4k^2s^2\Omega^2\{-\dot{u}_1(1+\alpha')+\nu' u_1\alpha'\}+\omega_p^2\,\mathscr{D}_{12}(\dot{u}_1+\nu' u_1)\\ &+2\Omega\{-\ddot{u}_2(1+\alpha')+\nu'\dot{u}_2(-1+\alpha')\}k^2s^2+\Omega\,\mathscr{D}_{12}(\ddot{u}_2+\nu'\dot{u}_2) = 0\end{aligned} \tag{5.16}$$

$$\begin{aligned}\mathscr{D}_{12}\,\mathscr{D}_b(\dot{u}_2+\nu u_2)&+k^2s^2\,\mathscr{D}_b\{\dot{u}_2(1+\alpha')+u_2\nu'\}\\ &+\omega_p^2\,\mathscr{D}_{12}\dot{u}_2+2\Omega k^2s^2\,\mathscr{D}_b\{u_1(1+\alpha')\}-\Omega\,\mathscr{D}_{12}\,\mathscr{D}_b u_1 = 0\end{aligned} \tag{5.17}$$

where

$$\alpha' = \alpha\exp(-\nu' t).$$

Equations (5.16) and (5.17) yield the dispersion relation for the collisionless case as

$$\begin{aligned}\left\{1+\frac{\omega_p^2}{k^2c^2-\omega^2}+\frac{k^2s^2(1+\alpha)}{4\Omega^2-\omega^2}\right\}&\left\{1-\frac{3k^2s^2(1+\alpha)}{\omega^2}-\frac{\omega_p^2}{\omega^2}+\frac{4k^2s^2(1+\alpha)\Omega^2}{\omega^2(4\Omega^2-\omega^2)}\right\}\\ &= \frac{\Omega^2}{\omega^2}\left\{1-\frac{2k^2s^2(1+\alpha)}{4\Omega^2-\omega^2}\right\}^2.\end{aligned} \tag{5.18}$$

In the opposite extreme case, when the pressure relaxation frequency $\nu' \to \infty$, the dispersion relation as obtained from equations (5.16) and (5.17) becomes

$$\begin{aligned}(\tfrac{5}{3}k^2s^2-\omega^2+i\omega\nu+\omega_p^2)\,&\{(k^2c^2-\omega^2)\,(\omega-i\nu)+\omega\omega_p^2\}\\ &+\omega\Omega^2(k^2c^2-\omega^2) = 0.\end{aligned} \tag{5.19}$$

Equation (5.19) is the usual expression for the coupled longitudinal–transverse waves with the effective value of $\gamma = \frac{5}{3}$.

6. Conclusions

The above analysis indicates that, in the presence of weak collisions, a plasma which has an anisotropic pressure may exhibit new collisional instabilities. The anisotropic pressure represents an anisotropic distribution of thermal energy, and the pressure relaxation mechanism pumps this energy from one direction into the other. This feeding of energy may give rise to the instability. It is seen that the momentum relaxation mechanism always induces damping; the pressure relaxation mechanism may increase or decrease the damping, depending upon the situation and the sign of the anisotropy parameter. The damping and phase-shift coefficients are explicitly calculated for certain cases and conditions are deduced for increasing oscillations to occur. For the transverse mode of propagation a high value of the anisotropy parameter is necessary for the instability to occur. The oscillations cease to increase as soon as the value of α falls below the required value. In the limit of infinite-pressure relaxation frequency the usual dispersion relations are again obtained and now the momentum relaxation mechanism alone contributes to the process of the damping of the wave.

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